Reliability Sensitivity Estimation of Complex Models: A Simulation-Based Approach

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1. Introduction

One of the objectives of reliability sensitivity analysis is to study the influence of probabilistic model parameters onto the reliability of a given structural system. In this context, the system parameters involved in the sensitivity analysis are modeled by a random vector whose joint probability distribution is explicitly known and dependent on a certain number of parameters. The effect of distribution parameters on the system reliability is obtained by calculating the partial derivative of the failure probability with respect to such parameters. Of practical importance is the reliability sensitivity analysis of medium/large nonlinear finite element models subject to stochastic excitation. This type of problems appears in a number of realistic situations related to for example, earthquake engineering, offshore engineering, wind engineering, etc. The determination of the variation in the reliability (or equivalently in the failure probability) due to changes in system parameters can provide useful information. For example it can be used to identify the most influential system parameters and it can provide an important insight on system failure for risk-based decision making, such as robust control or reliability-based design optimization.

2. Challenges

The main difficulty in estimating reliability sensitivity measures for nonlinear finite element models under stochastic excitation is that a large number of finite element model re-analyses are required over the space of system parameters. Consequently, the total computational demand may become excessive when the computational time for performing a single dynamic finite element analysis is significant. In this context, it is the objective of this contribution to integrate a model reduction technique into an efficient reliability sensitivity analysis of a class of medium/large nonlinear finite element models subject to stochastic excitation.

3. Methodology

For reliability purposes first excursion probabilities are used to characterize the level of safety of a structure. This probability measures the chances that uncertain responses exceed in magnitude prescribed thresholds within a specified time interval. The reliability sensitivity analysis is carried out by an approach recently proposed by the authors [1]. Such approach is a simple post-processing of an advanced sampling-based reliability analysis. Numerical results showed that on average the estimates generated by the proposed approach converge to the reference results obtained directly by Monte Carlo simulation. Validation calculations also indicated that the proposed approach is much more efficient than Monte Carlo, especially when estimating reliability sensitivity measures of systems with small failure probability. As previously pointed out, reliability sensitivity analysis of involved finite element models may be quite expensive from the numerical point of view. To cope with this difficulty, a model reduction technique is proposed to carry out the sensitivity analysis in a reduced space of generalized coordinates. In particular, a method known as component mode synthesis is implemented in the present formulation [2]. The method involves dividing the structure into a number of components obtaining reduced-order models of the components and then assembling a reduced-order model of the entire structure. If the division of the structure into components is guided by a certain parameterization scheme in terms of the system parameters, substantial computational savings can be achieved in estimating the reliability sensitivity measures. Moreover, the drastic reduction in computational efforts is obtained without compromising the accuracy of the sensitivity estimates.

4. Numerical Example

A three dimensional concrete bridge finite element model of about 11,000 degrees of freedom is considered to illustrate the effectiveness and computational efficiency of the proposed approach. The bridge model, shown in Figure (1), is curved in plan and it has a total length of 119.0m. It has 5 spans of lengths equal to 24.0m, 20.0m, 23.0m, 25.0m, and 27.0m, respectively, and four piers of 8m height that support the girder monolithically. Each pier is founded on an array of four piles of 35m height. The piers and piles are modeled as column elements of circular cross-section, while the deck cross-section is a box girder. The deck girder is modeled by beam and shell elements and it rests on each abutment through two rubber
bearings. These devices consist of layers of rubber and steel, with the rubber being vulcanized to the steel plates. A schematic representation of a rubber bearing is also shown in the figure as well as its nonlinear behavior. An analytical model that simulates measured restoring forces under bidirectional loadings is used in the present application to describe the nonlinear behavior of the rubber bearings. The model is based on a series of experimental tests conducted for real-sized rubber bearings [3]. The bridge structure is subjected to a ground acceleration which is modeled as a non-stationary stochastic process. In particular, a stochastic model which combines a point-source model with a velocity pulse model is considered in the present implementation [4]. The generation of the stochastic ground motions involves more than 3,000 random variables for the duration and sampling interval considered. Thus, the reliability sensitivity analysis is a high dimensional problem. A reduced-order model characterized by 88 generalized coordinates is constructed for this particular model. This represents a 99% reduction with respect to the full finite element model.

![Finite element model of the concrete bridge structure](image)

The performance of the bridge structure is characterized in terms of a failure event related to the maximum relative displacement between the top of the piers and their connections with the piles foundation. The reliability analysis is performed with respect to a series of system parameters. For example Fig. 2 shows the results of the reliability sensitivity analysis with respect to the diameter of the pier and pile elements in form of arrows indicating the magnitude and sign of the sensitivity. Twenty independent runs are shown in the figure. The sensitivity with respect to these parameters is negative. Thus, an increase in the value of these parameters decreases the probability of failure. In this case an increase in the diameter of the pier and pile elements tends to decrease the maximum relative displacement between the piers and the piles, as expected. The results also indicate that both parameters, that is, the diameter of the pier and pile elements have an important effect on the reliability of the system.

The computational effort for assembling the finite element model and obtaining its nonlinear dynamic response for a given set of system parameters is the fundamental factor for estimating the computation cost. In this regard, the proposed model reduction technique is quite effective. In fact, the use of the reduced-order model for estimating the reliability sensitivity measures results in a drastic reduction in the computational effort of almost two orders of magnitude.

![Sensitivity of the failure probability with respect to the diameter of the pier and pile elements: 20 independent estimations](image)

5. Conclusions

The numerical results show that the computational effort for estimating the reliability sensitivity measures of the reduced-order model is decreased significantly with respect to the full finite element model. This reduction is achieved without compromising the accuracy of the reliability sensitivity estimates. On the other hand, the reliability sensitivity analysis gives a valuable insight into the performance and reliability of complex structural systems such as finite element models under stochastic loads.

References


